Size of committees under outside influence*

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Abstract

The paper studies the impact of biased influence on the returns from increasing the size of a committee. We show that when the chance of preference misalignment between source of influence and voters is low, committee size is irrelevant and a small committee with the minimum number of just three voters generates the same probability of correct decision-making as any larger electorate. On the other hand, in settings with a high chance of preference misalignment, the smallest committee size needed to maximize this probability increases with the precision of voters private signals.

Keywords Committee decision · Public persuasion · Welfare

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1 Introduction

In this paper, we address the question to what extent information provided by a biased expert can influence voters and manipulate the outcome of collective decisions in a common value model of voting. In doing so, we revisit the Condorcet Jury Theorem and show that it is generally not robust to the introduction of strategic persuasion by an informed expert.

This question is motivated by the observation that inviting expert advisors to provide input into a committee is a common practice. It is often the case that owing to concerns regarding her reputation, an expert does not provide information which is incorrect, and this is common knowledge. However, it is entirely possible that the expert may have her personal biases that do (at least partially) conflict with the preferences of the committee members. While public knowledge about the expert's reputation concerns makes the expert better placed to credibly transmit any information in spite of her biases, this power of credibility may in turn enhance her ability to manipulate committee decisions. The expert may attempt this by suitably tailoring the content of the advice provided in an effort to manipulate committee voting behavior. This raises the interesting question of what is better for the voters: the availability of credible expert information (which may be subject to manipulation) or not receiving any additional information (and thus being shielded from expert manipulation). In particular, we ask what are the theoretical consequences of the presence of credible but strategically released information on committee members' ability to collectively make the correct decision.

To address this question we extend the common value voting model of **?** (with an odd number n of voters). In particular, we allow for strategic information transmission by a single informed expert (who does not participate in the collective decision but has free access to information) to many heterogeneous information receivers (the voters). There are two possible alternatives that the voters must choose from collectively: X or Y. The expert always prefers X. There is a state variable ω such that if ω is small enough then the voters also prefer X, but if ω is large enough they instead prefer Y. Therefore, the magnitude of ω determines the extent to which voters' and the expert's preferences are misaligned. The state ω is known to the expert but not to the voters. Each voter receives a signal $s_i \in X, Y$ which is informative about the state ω in the sense that with probability p the signal correctly reveals the right outcome given the state.

The expert chooses a persuasion communication strategy that generates a publicly observed message which divides the state space Ω into a finite partition $\{\Omega_k\}$ with a message Ω_k being interpreted as a statement that the true state ω is in subset Ω_k. Based on their signal s_i and the expert's public message individuals vote for X or Y and the outcome voted for by the majority of voters is chosen.

In this setting, we obtain the following results: First, we characterize the symmetric pure-strategy maximally informative equilibria in our setting with and without expert communication. Second, we show that the effect of introducing a strategic and biased expert has an ambiguous effect on the efficiency of the election outcome. That is, depending on the parameters of the model (the number of voters, the precision of their private signals p, and the degree of the conflict between the expert and voters), strategic persuasion by the expert can increase or decrease the probability that the collective decision yields the correct outcome. For instance, we show that strategic persuasion lowers the likelihood of a correct decision when there are many voters and their private signals are highly informative (this effect is reminiscent of herding behavior in situations in which publicly available imprecise information can cancel out private signals of individual players and thus, preclude efficient aggregation of information). However, more surprisingly, we show that in small electorates (i.e. fewer than five voters), and in particular when the conflict between expert and voters is small, the likelihood of a correct decision is non-monotonic in the precision of voters' private signals.

With this paper, we add to the literature on strategic persuasion by complementing the findings of **?**. In their framework, the receiver of expert information is a single uninformed decision-maker, rather than a group of voters who each also receive some relevant private information. **?** characterize sender-optimal persuasion strategies that (w.l.o.g.) take the form of an action recommendation by the expert, which is duly followed by the decision-maker. However, they only briefly discuss the possibility of extending their framework to either multiple receivers of information, or to a setting where the single decision-maker receives a private signal in addition to the expert's communication. Our setting, in contrast, combines both these scenarios, and shows that the expert's equilibrium persuasion strategy then is no longer always simply a recommendation as to how voters should vote. Instead, in our model, there are states of the world in which the expert conveys a public signal so that voters vote in line with their private signal. As mentioned before, our paper is related to **?** who also study the impact of public information on committee decisions. Although the the dissemination of public information is non-strategic in their framework, they show that there is no pure strategy Nash equilibrium where each voter votes sincerely and informatively when voters receive two private signals and any number of public signals. Moreover, the welfare implications of that work (for an arbitrary committee size n) is limited as the authors are focussed on obtaining informative voting in equilibrium so that the probability of correct decision making increases with n. The social value of public information has been a well addressed subject since the work of **?**. In a model with strategic complementarity, **?** show that public information can hurt social welfare only if agents also have access to independent sources of information. On the other hand, in the investment game of **?** public information necessarily improves welfare. Also, **?** show how welfare properties of public information depends not only on the form of strategic interaction but also on other external effects that determine the gap between equilibrium and efficient use of public information.[1] However in these papers, public information is non-strategic.

The remainder of the paper is organized as follows. In Section 2 we describe the model formally. Section 3 characterizes the optimal persuasion strategy of the expert and the subsequent voting behavior induced by it. Section 4 deals with information aggregation by comparing the scenarios where expert advice is available versus when it is not. We draw our conclusions in Section 5. The appendix (Section 6) contains most proofs.

2 The Model

Basic setting. There is a committee of voters $I = \{1, \cdots, n\}$ (where $n \geq 3$ and odd) who all have identical preferences over the two alternatives in $A \equiv \{X, Y\}$. Each voter must cast a vote in order to arrive at a collective choice from A. The voters' preferences over the set of alternatives A depend on an unknown state of the world $\omega \in \Omega \equiv [0,1]$. There is an expert with preferences different from those of the committee members who has free access to information about the unknown state.

Preferences. Voters all have the same state-dependent preference relation $\succ_\omega$ over A where, for some $0 < \omega_v < 1$, we have $X \succ_\omega Y$ if $\omega \leq \omega_v$ and $Y \succ_\omega X$ if $\omega > \omega_v$. This preference relation is represented by the utility function $u : A \times \Omega \to \mathbb{R}$ such that for $\underline{u}, \bar{u} \in \mathbb{R}, \underline{u} < \bar{u}$ we have:

[1] See also **?** and **?**, among others, for related works on impact of public information on social welfare.

$$u(X,\omega) = \begin{cases} \bar{u} & \text{if } \omega \le \omega_v, \\ \underline{u} & \text{otherwise;} \end{cases} \quad \text{and} \quad u(Y,\omega) = \begin{cases} \underline{u} & \text{if } \omega \le \omega_v, \\ \bar{u} & \text{otherwise.} \end{cases}$$

The expert strictly prefers X over Y in all states. This preference of the expert is represented by the utility function $u_m : A \times \Omega \to \mathbb{R}$ such that for $\underline{u}_m, \bar{u}_m \in \mathbb{R}$ with $\underline{u}_m < \bar{u}_m$ we have $u_m(X,\omega) = \bar{u}_m$ and $u_m(Y,\omega) = \underline{u}_m$ for all $\omega \in \Omega$.[2]

Information structure. The state of the world is modeled as a random variable, and agents' common prior over Ω is given by an atomless density function $f(\omega)$ with cumulative distribution function $F(\omega)$. The case $F(\omega_v) > 1/2$ will be referred to as *high likelihood of agreement* between voters and the expert, while *low likelihood of agreement* corresponds to $F(\omega_v) < 1/2$. Each voter $i \in I$ receives an i.i.d. private signal $s_i \in \{X,Y\} \equiv S$ with common precision $p \in (1/2, 1)$: $\mathbb{P}[s_i = X|\omega \le \omega_v] = \mathbb{P}[s_i = Y|\omega > \omega_v] = p$. Let $\boldsymbol{s} = (s_1, ..., s_n) \in S^n$ denote a signal-profile.

Communication. The expert can costlessly disseminate information about the true state of the world through his choice of *persuasion strategy*. Under any such strategy, the expert and the voters publicly receive information about the true state in the form of a specific 'range' of states to which the true one belongs. In particular, the expert announces his persuasion strategy prior to the realization of the true state. This strategy takes the form of an interval partition of the state space Ω such that an interval included in this partition is revealed to the agents if and only if the true state lies in that interval.[3] For any integer $k \ge 1$, let $\Omega^k = \{\Omega_1^k, \cdots, \Omega_k^k\}$ denote a k-element interval partition of Ω chosen and announced by the expert. A *public signal* generated by a persuasion strategy is therefore an interval $\Omega_t^k \subseteq \Omega$, where $t \in \{1, \ldots, k\}$. Given a signal Ω_t^k, the agents' posterior density function is $f(\omega|\Omega_t^k)$, which is obtained using Bayes rule.

Voting. After receiving observing the information revealed through the expert's chosen persuasion strategy and their respective private signals, voters cast their votes simultaneously. Given a persuasion strategy Ω^k, a *(pure) voting strategy* for voter $i \in I$ is a function $v_i : \Omega^k \times S \to A$ that maps the a public signal $\Omega_t^k \in \Omega^k$ and the private signal $s_i \in S$ to a *vote* $v_i(\Omega_t^k, s_i) \in A$. Let $\mathscr{V}$ be the set of all possible voting strategies of a voter, and denote by $\boldsymbol{v}(\Omega_t^k, \boldsymbol{s})$ the vote-profile $(v_1(\Omega_t^k, s_1), ..., v_n(\Omega_t^k, s_n))$. In order to capture the way in which voters' individual votes are aggregated into a collective decision, we introduce the notion of a *majoritarian committee decision function* $\delta : A^n \to A$ that maps a vote profile $\boldsymbol{v} \in A^n$ to an outcome $\delta(\boldsymbol{v}) \in A$ such that $\delta(\boldsymbol{v}) = X$ if and only if $|\{i \in I : v_i = X\}| \ge \frac{n+1}{2}$.

Equilibrium. We focus on symmetric (pure strategy) Perfect Bayesian equilibria of the voting continuation game where each voter follows a 'rational voting strategy' - a term coined by **?**. Under such a strategy, a voter votes in favor of the alternative that maximizes his expected utility after having made full use of his available information, which consists of the public signal generated by the expert's persuasion strategy, the voter's private signal, and any inference about other voters' signals that can be drawn from the fact that his vote affects the outcome only in events where he is pivotal. We call an equilibrium in which all voters use a rational voting

[2] Our results remain qualitatively intact in a more general environment where in some states the expert prefers alternative Y.

[3] Explain here that it is w.l.o.g. to consider communication strategies of this form!!!

strategy a *rational voting equilibrium* (RVE). Given a persuasion strategy Ω^k, a vote-profile $\boldsymbol{v}(\Omega_t^k, s)$ constitutes a RVE of the voting continuation game if for every $i \in I$, $\Omega_t^k \in \Omega^k$, $\boldsymbol{s} \in S^n$, and all $\hat{v}_i \in \mathscr{V}$, we have:

$$\int_{\Omega_t^k} \left(\sum\nolimits_{\boldsymbol{s}_{-i} \in S^{n-1}} \mathbb{P}[\boldsymbol{s}_{-i}|\omega, \Pi_i)]u(\delta(\boldsymbol{v}(\Omega_t^k, \boldsymbol{s})), \omega)\right) f(\omega|\Omega_t^k, s_i) d\omega \geq$$
$$\int_{\Omega_t^k} \left(\sum\nolimits_{\boldsymbol{s}_{-i} \in S^{n-1}} \mathbb{P}[\boldsymbol{s}_{-i}|\omega, \Pi_i]u(\delta(\hat{v}_i, \boldsymbol{v}_{-i})), \omega)\right) f(\omega|\Omega_t^k, s_i) d\omega,$$

where Π_i denotes the event that voter i is pivotal and $\boldsymbol{v}_{-i}$ is the profile of votes across all voters other than i. We note here that for every persuasion strategy Ω^k and for each $\omega \in \Omega$, there is always a unique symmetric RVE in the voting continuation game. Given this, we proceed to define the equilibrium of the full game. As in **?**, a persuasion strategy constitutes an equilibrium of the full game if and only if it maximizes the expert's ex-ante expected payoff. Take a strategy-pair (Ω^k, v) s.t. the symmetric voting-strategy v is the RVE of the continuation voting game given the persuasion strategy Ω^k. Then (Ω^k, v) is an equilibrium of the full game if for all other pairs $(\Omega^{\hat{k}}, \hat{v})$ such that $\hat{v}$ is the RVE of the continuation voting game given the persuasion strategy $\Omega^{\hat{k}}$, we have:

$$\int_{\Omega} \left(\sum\nolimits_{\boldsymbol{s} \in S^n} \mathbb{P}[\boldsymbol{s}|\omega]u_m(\delta(v(\Omega^k, \boldsymbol{s})), \omega)\right) f(\omega) d\omega \geq$$
$$\int_{\Omega} \left(\sum\nolimits_{\boldsymbol{s} \in S^n} \mathbb{P}[\boldsymbol{s}|\omega]u_m(\delta(\hat{v}(\Omega^{\hat{k}}, \boldsymbol{s})), \omega)\right) f(\omega) d\omega.$$

There may be multiple equilibria of the full game, but since all equilibria will be payoff-equivalent for the expert (and therefore payoff equivalent for the voters), we shall consider henceforth only the coarsest equilibrium persuasion strategy.

Information aggregation. We evaluate the performance of collective decision-making under biased persuasion by studying the *ex ante* expected probability with which voters collectively choose the right alternative (based on their preferences). Note that this evaluation criterion is equivalent to the maximization of the *ex ante* expected utility of a representative voter which, for a given strategy-pair (Ω^k, v), is given by:

$$U(\Omega^k, v) = \int_{\Omega} \left(\sum\nolimits_{\boldsymbol{s} \in S^n} \mathbb{P}[\boldsymbol{s}|\omega]u(\delta(v(\Omega^k, \boldsymbol{s})), \omega)\right) f(\omega) d\omega.$$

3 Equilibrium persuasion

3.1 No persuasion

As a benchmark, we characterize first voters' equilibrium behavior in the absence of persuasion by an expert. The details of this characterization will be useful below when we study RVE in the presence of expert communication. Note that we restrict attention to symmetric voting equilibria, and suppose all voters $j \in I$, $j \neq i$ follow the same voting strategy v. Denote the profile of votes across these $n-1$ voters by the vector $\boldsymbol{v}(\boldsymbol{s}_{-i})$, where:

$$\boldsymbol{v}(\boldsymbol{s}_{-i}) \equiv (v(s_1), \ldots, v(s_{i-1}), v(s_{i+1}), \ldots, v(s_n)).$$

Then voter i's interim expected utility from submitting a vote $v_i \in A$ conditional on his private signal s_i is given by:

$$\int_{\Omega} \left(\sum_{\boldsymbol{s}_{-i} \in S^{n-1}} \mathbb{P}[\boldsymbol{s}_{-i}|\omega] u\left(\delta\left(v_i, \boldsymbol{v}\left(\boldsymbol{s}_{-i}\right)\right), \omega\right) \right) f(\omega|s_i) d\omega.$$

As the collective decision is made according to the simple majority rule, voter i's vote affects the outcome of the election only if the remaining $n-1$ votes are split equally across the two alternatives. To ease notation, we define as $n_X(\boldsymbol{v}(\boldsymbol{s}_{-i}))$ the number of votes for alternative X cast by the the $n-1$ voters other than i. Voter i is pivotal in the sense that his vote affects the outcome if and only if $n_X(\boldsymbol{v}(\boldsymbol{s}_{-i})) = (n-1)/2$. We introduce the following notation to describe this event:

$$\Pi_i(v) \equiv \{\boldsymbol{s}_{-i} \in S^{n-1} : n_X(\boldsymbol{v}(\boldsymbol{s}_{-i})) = (n-1)/2\}.$$

With this notation, we can write as follows the difference in voter i's ex post utility from voting for X rather than Y in any state of the world $\omega \in \Omega$:

$$|u\left(\delta\left(X, \boldsymbol{v}\left(\boldsymbol{s}_{-i}\right)\right), \omega\right) - u\left(\delta\left(Y, \boldsymbol{v}\left(\boldsymbol{s}_{-i}\right)\right), \omega\right)| = \begin{cases} \bar{u} - \underline{u} & \text{if } n_X(\boldsymbol{v}(\boldsymbol{s}_{-i})) = \frac{n-1}{2} \\ 0 & \text{otherwise.} \end{cases}$$

This, together with the fact that voters' signal are conditionally independent, allows us to express the difference in interim expected utility as follows:

$$\sum_{\boldsymbol{s}_{-i} \in \Pi_i(v)} (\bar{u} - \underline{u}) \mathbb{P}[\boldsymbol{s}_{-i}] \left(2F(\omega_v|s_i, \boldsymbol{s}_{-i}) - 1\right).$$

Since we restrict attention to symmetric pure voting strategies v, any signal-profile $\boldsymbol{s}_{-i} \in \Pi_i(v)$ must consist of $(n-1)/2$ X-signals and $(n-1)/2$ Y-signals. Therefore, the set $\Pi_i(v)$ contains $\binom{n-1}{\frac{n-1}{2}}$ signal-profiles. As all the private signals are independent conditional on the state of the world, every profile $\boldsymbol{s}_{-i} \in \Pi_i(v)$ arises with the following probability:

$$\mathbb{P}[\boldsymbol{s}_{-i}] = \mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v) + \mathbb{P}[\boldsymbol{s}_{-i}|\omega > \omega_v](1 - F(\omega_v)),$$

where:

$$\mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] = \mathbb{P}[\boldsymbol{s}_{-i}|\omega > \omega_v] = p^{\frac{n-1}{2}} (1-p)^{\frac{n-1}{2}}.$$

This implies that being pivotal reveals *no* information to voters, so that$F(\omega_v|s_i, \boldsymbol{s}_{-i}) = F(\omega_v|s_i)$ for every $\boldsymbol{s}_{-i} \in \Pi_i(v)$. To see this, note that:

$$\begin{aligned} F(\omega_v|s_i, \boldsymbol{s}_{-i}) &= \frac{\mathbb{P}[s_i, \boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v)}{\mathbb{P}[s_i, \boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v) + \mathbb{P}[s_i, \boldsymbol{s}_{-i}|\omega > \omega_v](1 - F(\omega_v))} \\ &= \frac{\mathbb{P}[s_i|\omega \leq \omega_v] \mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v)}{\mathbb{P}[s_i|\omega \leq \omega_v] \mathbb{P}[\boldsymbol{s}_{-i}|\omega \leq \omega_v] F(\omega_v) + \mathbb{P}[s_i|\omega > \omega_v] \mathbb{P}[\boldsymbol{s}_{-i}|\omega > \omega_v](1 - F(\omega_v))} \\ &= \frac{\mathbb{P}[s_i|\omega \leq \omega_v] F(\omega_v)}{\mathbb{P}[s_i|\omega \leq \omega_v] F(\omega_v) + \mathbb{P}[s_i|\omega > \omega_v](1 - F(\omega_v))} = F(\omega_v|s_i). \end{aligned}$$

It is now easy to characterize the optimal voting-behavior of any voter i. As $\bar{u} > \underline{u}$ by assumption:

- vote for alternative X if the interim utility difference is positive, which is the case iff $F(\omega_v|s_i) > 1/2$;
- vote for alternative Y if the interim utility difference is negative, which is the case iff $F(\omega_v|s_i) < 1/2$.

Note that if $F(\omega_v|s_i) = 1/2$, voter i is indifferent between voting for X or Y. For simplicity, we shall assume in this case that any voter i acts in line with the expert's preference, which is to vote for alternative X.

Proposition 1. *In the absence of public persuasion by an expert, there is a unique symmetric RVE. For every voter $i \in I$ and all $s_i \in S$:*

1. *if $F(\omega_v) > 1/2$: $v(s_i) = s_i$ for $p > F(\omega_v)$, and $v(s_i) = X$ otherwise;*
2. *if $F(\omega_v) < 1/2$: $v(s_i) = s_i$ for $p > 1 - F(\omega_v)$, and $v(s_i) = Y$ otherwise.*

The proof is in Section 6.1 in the appendix. It is worth noting that, according to Prop. 1, there are only two scenarios in which the unique RVE of the game without public persuasion involves committee members voting in line with their private signals (namely if either $p > F(\omega_v) > 1/2$, or $1/2 > F(\omega_v) > 1 - p$). Such a strategy is called *informative voting* by **?**. It is only in these two scenarios that the Condorcet Jury Theorem prevails, whereby the probability of a correct committee decision approaches 1 as the number n of voters becomes arbitrarily large. We will see in what follows that public persuasion may distort the collective decision even for these values of the model parameters.

3.2 Persuasion under high likelihood of agreement

We begin with the case of high likelihood of agreement between the expert and the voters: $F(\omega_v) > 1/2$. The following proposition provides a full characterization of the equilibrium in this case:

Proposition 2. *Let $F(\omega_v) > 1/2$. Then the unique coarsest equilibrium features a binary persuasion strategy with threshold $\omega^* > \omega_v$ s.t. $\Omega_1^2 = [0, \omega^*]$ and $\Omega_2^2 = (\omega^*, 1]$. In particular:*

1. *if $p < F(\omega_v)$, then $\omega^* = 1$ and $v_i(\Omega_1^1, s_i) = X$ for all $s_i \in S$ (i.e. persuasion yields no information);*
2. *if $p > F(\omega_v)$, then $F(\omega^*) = F(\omega_v)/p$ (i.e. $\omega^* > \omega_v$), $v_i(\Omega_1^2, s_i) = X$ for all $s_i \in S$ and $v_i(\Omega_2, s_i) = Y$ for all $s_i \in S$.*

The proof is in Section 6.2 in the appendix. To gain some intuition for the result in Prop. 1, note that there are only two candidates for the coarsest equilibrium persuasion strategy: First, the binary strategies described in Prop. 1 (where, regardless of signal, voters vote for X if Ω_1^2 is announced, and vote for Y if Ω_2^2 is announced), and second, the class of ternary persuasion strategies Ω^3 where voters vote in line with their private signals when the expert submits an intermediate report $\Omega_2^3 = [\alpha, \beta]$ where $0 \leq \alpha < \omega_v \leq \beta \leq 1$ (otherwise, they all vote for the same alternative regardless of signal - namely X if $\Omega_1^3 = [0, \alpha]$ is reported, and Y

if $\Omega_3^3 = [\beta, 1]$ is reported). The proof shows that the coarsest equilibrium persuasion strategy is unique and is binary in nature. When $p < F(\omega_v)$ it follows from Prop. 1 that whenever no information is conveyed by the expert, every member votes for X irrespective of his private signal its strength is sufficiently low (i.e. $p < F(\omega_v)$). Since the expert prefers alternative X in all states of the world, this is the ideal scenario for him and therefore he chooses the persuasion strategy that does not transmit any information. This explains part (a) of Prop. 1. However, if voters' signals are sufficiently informative (i.e. $p > F(\omega_v)$), then Prop. 1 implies that in the absence of any information from the expert, voters vote in line with their private signals. This scenario is suboptimal for the expert who, in equilibrium, provides information Ω_1^2 about the state of the world so that voters choose X irrespective of their private signals when they hear this report. When, instead, Ω_2^2 is declared by the expert, the voters always choose the expert's least preferred alternative Y. In equilibrium, the expert's persuasion strategy maximizes the length of the interval Ω_1^2 for which voters are willing to vote for X regardless of their private signals. To see this, observe that for voters to choose X regardless of signal, the expert's report Ω_1^2 must provide information that is sufficiently strongly in favor of X (i.e. the likelihood that a state greater than ω_v has generated the report Ω_1^2 must be sufficiently low) so that the voters choose X even when they receive a private signal of Y.[4]

Remark 1. *The threshold ω^* that describes the unique equilibrium persuasion strategy is a decreasing function of p. This implies that more informed voters receive more accurate public information from the expert.*

3.3 Persuasion under low likelihood of agreement

We now consider the scenario where the likelihood of agreement between the expert and the voters is low: $F(\omega_v) < 1/2$. In this case, the characterization of the expert's equilibrium persuasion strategy depends not only on the likelihood of agreement, but also on the interplay of $F(\omega_v)$ with the other model parameters, namely the number of voters n and the signal precision p. In particular, what will matter for the expert's choice of persuasion strategy is the likelihood that in the event of disagreement, committee members voting in line with their private signals choose collectively the expert's favored outcome X. Formally:

$$\mathbb{P}[\delta(\boldsymbol{s}) = X | \omega > \omega_v] = 1 - J_n(p),$$

where $J_n(p) \equiv \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} p^j (1-p)^{n-j}$ is the probability that more than half of the voters receive the correct private signal given the true state of the world. The condition stated in the following definition provides us with a threshold that determines whether the likelihood $\mathbb{P}[\delta(\boldsymbol{s}) = X | \omega > \omega_v]$ is deemed high or low by the expert. This, in turn, determines his optimal persuasion strategy.

Definition 1. *In the event of disagreement between the expert and the voters, the likelihood that the expert's favorite outcome X is chosen when committee members vote in line with their private signals is* ***high*** *if:*

$$1 - J_n(p) > \frac{F(\omega_v)/(1-F(\omega_v))}{(p/(1-p)) - 1}. \tag{1}$$

[4] Note that the expert could, in principle, achieve the same result with an alternative communication strategy: He can send a message which indicates that his least preferred alternative is the correct choice some (but not all) of the time. Whenever the expert remains silent voters will infer that with sufficiently high probability, the state is s.t. the expert's favorite alternative is the correct choice. A similar persuasion strategy is used in **?** for a single decision maker.

The threshold on the right-hand side of (1) is an odds ratio, where the numerator contains the odds in favor of agreement, and the denominator contains the scaled conditional odds in favor of a correct signal.[5] Note that this threshold is always positive, so that a necessary condition for (1) is that the odds ratio be strictly below 1. This, as the reader can easily verify, is the case if and only if the odds of an *incorrect* signal are lower than the probability of disagreement: $1-F(\omega_v) > (1-p)/p$. From this inequality it follows immediately that condition (1) fails when the private signal becomes perfectly informative (i.e. $p \to 1$). The same is true when the number of voters is large because $J_n(p) \to 1$ as $n \to \infty$.

The following result characterizes the expert's equilibrium persuasion strategy and the resulting equilibrium voting behavior of the committee members:

Proposition 3. *If $F(\omega_v) < 1/2$, the unique coarsest equilibrium features a binary persuasion strategy with threshold $\omega^* \in (0,1)$ s.t. $\Omega_1^2 = [0,\omega^*]$ and $\Omega_2^2 = (\omega^*,1]$. In particular:*

1. *If $p > 1-F(\omega_v)$ and furthermore:*
   (a) *condition (1) in Definition 1 holds, then ω^* is given by $F(\omega^*) = (F(\omega_v)-(1-p))/p$, which implies $\omega^* < \omega_v$. The voters' symmetric equilibrium strategy is $v_i(\Omega_1^2, s_i) = X$ for all $s_i \in S$ and $v_i(\Omega_2^2, s_i) = s_i$;*
   (b) *condition (1) in Definition 1 is violated, then ω^* is given by $F(\omega^*) = F(\omega_v)/p$, which implies that $\omega^* > \omega_v$. The voters' symmetric equilibrium strategy is $v_i(\Omega_1^2, s_i) = X$ and $v_i(\Omega_2^2, s_i) = Y$ for all $s_i \in S$.*

2. *If $p < 1-F(\omega_v)$ and furthermore:*
   (a) *$n \geq 5$, then the equilibrium persuasion strategy and voting behavior is as in 1.(b);*
   (b) *$n = 3$, then there exists a $\bar{p} \in (1/2,1)$ s.t. for all $p < \bar{p}$ the equilibrium persuasion strategy and voting behavior are the same as in 1.(b); for $p > \bar{p}$ the threshold ω^* is given by $F(\omega^*) = F(\omega_v)/(1-p)$, which implies $\omega^* > \omega_v$. The voters' symmetric equilibrium strategy is $v_i(\Omega_1^2, s_i) = s_i$ and $v_i(\Omega_2^2, s_i) = Y$ for all $s_i \in S$.*

The proof is in Section 6.2 in the appendix. To gain some intuition for the results in Prop. 2, recall first that the expert is happy whenever voters collectively choose X (his favorite alternative). In particular, he would like voters to choose X in the event of disagreement. If condition (1) in Definition 1 holds, the expert is willing to let voters follow their respective private signals for a large stretch of the state space (i.e. for all $\omega > \omega^*$, where $\omega^* < \omega_v$) because the chance that voters collectively choose X in the event of disagreement (i.e. $1-J_n(p)$) is sufficiently high.

4 Information aggregation

In this section, we ask how the likelihood of making the correct collective decision is affected by the expert's communication, relative to a scenario without persuasion. Voters' preferences form the benchmark for deciding what constitutes the correct decision, because the expert considers alternative X to be the correct choice in every state of the world. We then briefly consider

[5] Note that the numerator odds take values in $\mathbb{R}_{++}$, while the odds $p/(1-p)$ take values in $(1,\infty)$ due to our assumption that private signals are informative (i.e. $p > 1/2$). In order to be able to form a meaningful odds ratio, the denominator odds have to be scaled so as to also take values in $\mathbb{R}_{++}$.

the question of optimal committee size in the presence of a biased expert, where optimality refers to the highest possible *ex ante* probability that the voters collectively choose the right alternative.

4.1 Information aggregation under high likelihood of agreement

We begin with the case of high likelihood of preference alignment between the expert and the voters: $F(\omega_v) > 1/2$. Our results below show that whether expert persuasion harms information aggregation or not depends on the size n of the electorate. We will see that seven or more voters, persuasion unambiguously reduces the probability of a correct collective decision relative to a scenario without expert persuasion. However, with three and with five voters, the situation is more nuanced in that the result will depend on the interplay of signal precision p and the preference bias of the expert. In particular, for an intermediate level of signal precision, the probability of a correct decision will be higher *with* persuasion, while for low and high levels of signal precision it will be higher *without* persuasion. These observations are made precise in the following proposition.

Proposition 4. *Public persuasion under high likelihood of agreement has the following effect on the probability that voters collectively choose the correct alternative given the true state of the world:*

1. *If $p < F(\omega_v)$, the likelihood that voters collectively choose the correct alternative is unaffected by the expert's presence.*
2. *If $p > F(\omega_v)$ and furthermore:*
   (a) *if $n \geq 7$, the likelihood of choosing the correct alternative is higher in the absence of persuasion;*
   (b) *if $n < 7$, there is a threshold $q_n \in (1/2, p)$ s.t. if $F(\omega_v) > q_n$, the probability of making the correct decision is higher in the absence of persuasion. If, instead, $F(\omega_v) < q_n$, there exists an interval $[\hat{p}_n^F, \tilde{p}_n^F]$ with $F(\omega_v) < \hat{p}_n^F < \tilde{p}_n^F < 1$ s.t. for all $p \in [\hat{p}_n^F, \tilde{p}_n^F]$, the likelihood of choosing the correct alternative is no lower* with *expert persuasion, and is strictly higher* with *expert persuasion for all p in the interior of this interval. For any p outside this interval, the probability of making the right choice is higher* without *expert persuasion.*

Proof of items 1. and 2.(a) of Proposition 4. First consider the case $p < F(\omega_v)$. Item 1. of Prop. 4 follows immediately from Prop. 1 and Prop. 2.(a). Next, consider the case $p > F(\omega_v)$. From Prop. 1 it follows that, in the absence of an expert, voters cast their votes in line with their respective private signals. As a result, they choose the correct alternative with probability $J_n(p)$, which converges to 1 as the size of the electorate gets large. By Prop. 2.(b) we know that, in the presence of an expert, the equilibrium (Ω^2, v) induces the following probability of making the correct decision: $1 - F(\omega_v)(1-p)/p$. Thus, expert persuasion harms information aggregation iff:

$$F(\omega_v) > \frac{p}{1-p}(1 - J_n(p)) \equiv G_n(p). \tag{2}$$

Note that for all n, G_n is continuously differentiable, that $G_n(0) = G_n(1) = 0$, and that $G_n(1/2) = 1/2$. The following lemma helps us establish item 2.(a) of Prop. 4:

Lemma 1. *For all $n \geq 7$, the function $G_n(p)$ is strictly decreasing for all $p \in [1/2, 1)$.*

The proof of this lemma is in Section 6.4 in the Appendix, and Fig. 1 provides a graphical illustration of G_n for select $n \geq 7$.

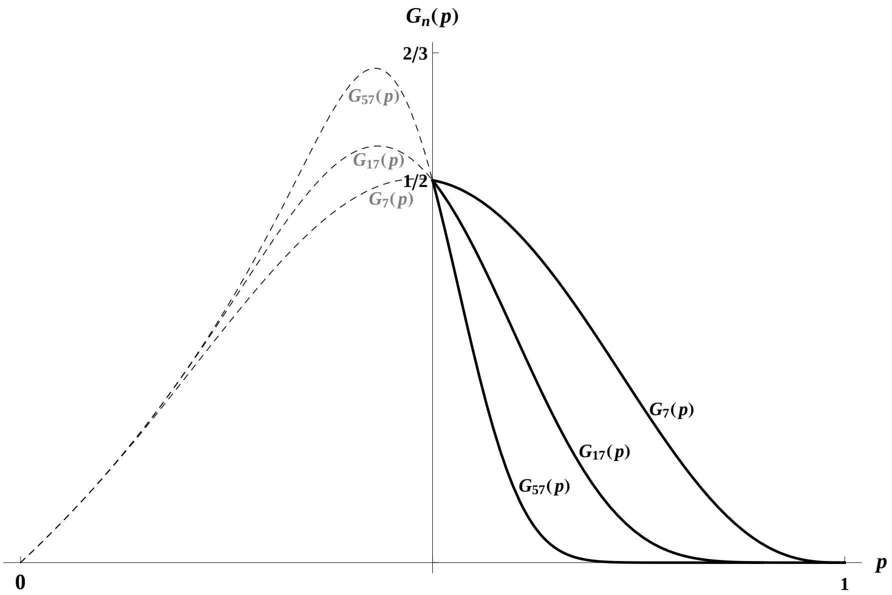

Figure 1: Three illustrations of the function G_n defined in (2)

As we have $F(\omega_v) > 1/2$ in item 2. of Prop. 4, the condition $F(\omega_v) > G_n(p)$ holds for all $p \in (1/2, 1)$ if $n \geq 7$. This establishes that expert persuasion impedes information aggregation in electorates with seven or more voters. □

The proof of item 2.(b) of Prop. 4 is relegated to Section 6.7 in the Appendix. However, the idea behind the proof can be readily understood by looking at Fig. 2, which shows that the functions G_3 and G_5 each have a unique maximum in the interior of the range $(1/2, 1)$. Thus, if $F(\omega_v)$ exceeds this maximum, then expert persuasion harms information aggregation. If, instead, $F(\omega_v)$ is below this maximum, then there exists an interval whose boundaries arise from the intersection of the horizontal line at $F(\omega_v)$ with the graph of G_n ($n = 3, 5$). For signal precision p within this interval, expert persuasion - despite its bias - enhances the probability of a correct collective decision relative to the benchmark of no persuasion.

In the remainder of this section, we provide some intuition for the results in Prop. 4. When the signal strength is low (i.e. $p < F(\omega_v)$) each member votes for X in all states of the world irrespective of his private signal and irrespective of the number of voters (see Prop. 1). This rationalizes the expert's decision not to transmit any information to the voters. As a result, the probability of making the correct decision is the same whether or not an expert is present.

But when the signal strength is high (i.e. $p > F(\omega_v)$), the probability of making the correct decision is higher without persuasion for a sufficiently large electorate (i.e. $n \geq 7$). The reason is as follows: in the presence of an expert, the information provided is s.t. voters opt for the wrong alternative X when $\omega \in (\omega_v, \omega^*]$, regardless of how many voters there are. If, instead, there is no expert and voters vote in line with their signals, the chance of a wrong decision in states $\omega \in (\omega_v, \omega^*]$ diminishes as the size of the electorate goes up. A noteworthy feature of our result is that the critical size of the electorate is $n = 7$.

Now consider the case of $n = 3$ or $n = 5$ voters. In these cases, the probability that voters make the right choice when voting in line with their private signals is low. It is here that the

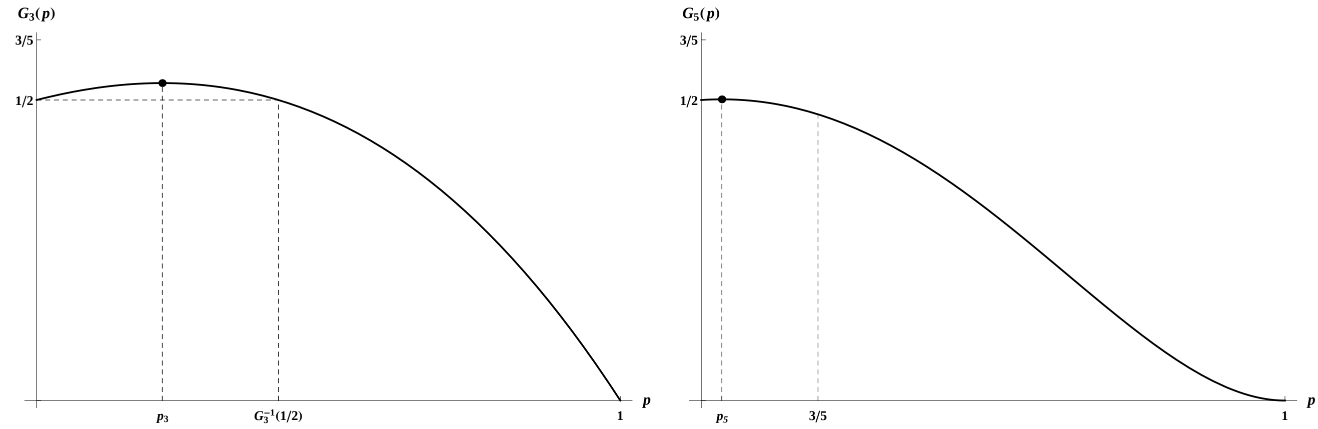

Figure 2: Illustration of the functions G_3 and G_5 for values $p \in [1/2, 1]$

analysis gets interesting due to the non-monotonicity in the ranking of the probabilities of a correct decision with and without the expert: for $p > F(\omega_v)$, the length of the interval $(\omega_v, \omega^*]$ over which the expert can manipulate voters into choosing the wrong decision *decreases* with p. I.e. for low p there is a large range of states for which voters are being manipulated, meaning that the probability of a correct decision is higher without the expert. If, instead, the signal precision p is very high, the probability that voters will collectively choose the correct alternative without persuasion is anyway high in all states - even in such small electorates. Only in an intermediate range of p is it worthwhile to suffer the expert's manipulation in exchange for his help in choosing the correct alternative all states except $\omega \in (\omega_v, \omega^*])$.

Finally, we briefly turn to the question of optimal committee size under biased persuasion. Our result in Section 3.2 gives rise to the following insight:

Corollary 1. *Under biased expert persuasion in settings with high likelihood of agreement, the probability of a correct collective decision is constant for all $n \geq 5$.*

Proof of Corollary 1 By Prop. 2, we know that all voters use the same signal-independent voting strategy. This results in a constant probability of making the correct choice (namely $F(\omega_v)$ in the scenario where $p < F(\omega_v)$, and $1 - F(\omega_v)(1-p)/p$ if $p > F(\omega_v)$) regardless of the size of the electorate. □

Thus, when the chance of preference misalignment between expert and voters is low, committee size is irrelevant and a small committee with the minimum number of just three voters generates the same probability of correct decision-making as any larger electorate.

4.2 Information aggregation under low likelihood of agreement

When the likelihood of preference misalignment is large ($F(\omega_v) < 1/2$), our equilibrium characterization in Section 3.3 hinges on condition (1) in Definition 1. Therefore, the impact of persuasion on the probability of correct decision-making will also depend on this condition, and particularly so when the signal precision exceeds the likelihood of disagreement: $p > 1 - F(\omega_v)$. The following proposition states our results formally:

Proposition 5. *Public persuasion under low likelihood of agreement has the following effect on the probability that voters collectively choose the correct alternative given the true state of the world:*

1. *If* $1-F(\omega_v) < p$ *and if condition (1) in Definition 1 is violated, then the likelihood that voters collectively choose the correct alternative is higher with expert persuasion iff* $F(\omega_v) < G_n(p)$. *If, instead, condition (1) is violated, then the probability of a correct collective decision is always higher with persuasion.*

2. *If* $p < 1-F(\omega_v)$ *expert persuasion always enhances the probability of a correct collective decision.*

Proof of Proposition 5. First consider $p > 1-F(\omega_v)$. By Prop. 1 the probability of a correct collective decision is $J_n(p)$. In the presence of the expert, Prop. 3.1.(b) implies that if condition (1) is violated, the probability of a correct collective decision is $1-F(\omega_v)(1-p)/p$. Thus, expert persuasion harms information aggregation iff $F(\omega_v) < G_n(p)$, where $G_n(p)$ is given in (2). Therefore, using arguments analogous to those in the proof of item 2. of Prop. 4 we can establish for which model parameters n, p, and $F(\omega_v)$ expert persuasion harms or aides information aggregation. Now suppose that condition (1) holds, in which case Prop. 3.1.(a) implies that the probability of a correct collective decision is $1-(1-J_n(p))(1-F(\omega_v))/p$. It is straightforward to verify that this probability exceeds the probability $J_n(p)$ with which voters make a correct decision in the absence of persuasion.

Now consider $p < 1-F(\omega_v)$. By Prop. 1 the probability of a correct decision in the absence of persuasion is $1-F(\omega_v)$, regardless of the number of voters. In the presence of the expert, Prop. 3.2.(a) implies that for $n \geq 5$ voters, the probability of a correct collective decision is $1-F(\omega_v)(1-p)/p$. Thus, expert persuasion always harms information aggregation. The same results follows by Prop. 3.2.(b) for $n = 3$ voters and signal precision $p < \bar{p}$. If, instead, $n = 3$ and $p > \bar{p}$, then Prop. 3.2.(b) implies that the probability of a correct collective decision is $1-F(\omega_v)(1-J_n(p))/(1-p)$. It is straightforward to verify that this probability exceeds the probability $J_n(p)$ with which voters make a correct decision in the absence of persuasion. □

We now provide some intuition for the results in Prop. 5. Consider item 1. and suppose that condition (1) is violated (which is the case for large n regardless of the values of p and $F(\omega_v)$). In this case, expert persuasion is harmful. This is because in the expert's presence, the probability of a correct decision is invariant to the number of voters as each of them follows a signal-invariant voting strategy that results in a unanimous decision. In the expert's absence, voters vote according to their private signals, which implies that in large electorates a large number of private signals is being aggregated into a collective decision. This ensures that the correct decision is made with high probability. If, instead, the size of the electorate is small, the desirability of expert communication hinges on the 'relative size' of the set of states for which decision-making is improved owing to any information provided by the expert and the set of states in which voters are manipulated into voting for the wrong alternative.

Now consider the case where condition (1) holds. By Prop. 3.1.(a), expert persuasion makes voters vote for their favorite alternative X regardless of their respective private signals in all states $\omega \in [0, \omega^*]$, where $\omega^* < \omega_v$. In contrast, without expert persuasion voters would cast their vote in line with their private signals in these states. In all remaining states, voters vote according to their private signals, which they would do also in the absence of the expert. It is therefore easy to see that expert persuasion enhances the chances that the electorate makes the correct decision.

Finally, in item 2. of Prop. 5, the signal precision is so low that in the expert's absence voters vote for Y irrespective of their private signals. Given that the likelihood of preference misalignment is high, the expert's power to manipulate voters into voting for the wrong alternative is limited and is outweighed by his help in guiding voters to the correct alternative when the true state is in $[0, \omega_v]$.

To conclude this section, we turn to the question of optimal committee size under biased persuasion.

Corollary 2. *Under biased expert persuasion in settings with low likelihood of agreement, the probability of a correct collective decision is maximized by an electorate of the following size:*

1. *if $p > 1 - F(\omega_v)$: $n \geq n^*$, where n^* is the smallest integer for which condition (1) in Definition 1 is violated;*
2. *if $p < 1 - F(\omega_v)$ and furthermore:*
   (a) *if $p < \bar{p}$ (where $\bar{p}$ is defined in item 2.(b) of Prop. 3): $n \geq 3$;*
   (b) *if $p > \bar{p}$: $n \geq 5$.*

Proof of Corollary 2 First consider the case $p > 1 - F(\omega_v)$. If condition (1) is violated (which, for given p and $F(\omega_v)$, it will be for any $n \geq n^*$), then the probability of making the correct collective choice is constant and will therefore not rise if additional voters are added: $1 - F(\omega_v)(1-p)/p$. Now contrast such a 'large' committee with a 'small' one where the number of voters is low enough so that condition (1) holds (i.e. $n < n^*$). In this case, the probability of making the correct collective choice is $1 - (1 - J_n(p))(1 - F(\omega_v))/p$, which increasing in n for all $n < n^*$. Note that the probability of correct decision-making is *higher* in a 'large' committee than a 'small' one iff:

$$1 - J_n(p) > \frac{F(\omega_v)(1-p)}{1 - F(\omega_v)}. \tag{3}$$

To see that this inequality holds for all 'small' committees of size $n < n^*$, note that we can rewrite condition (1) in Definition 1 as follows: $(1 - J_n(p))(2p-1) > F(\omega_v)(1-p)/(1 - F(\omega_v))$. As this inequality holds by definition for any 'small' committee of size $n < n^*$, it is immediately obvious that the inequality in (3) is satisfied, which implies that the probability of correct decision-making is higher in 'large' committees of size $n \geq n^*$.

Now consider the case of $p < 1 - F(\omega_v)$. By item 2. of Prop. 3 the probability of correct decision-making with $n \geq 5$ voters, and also with $n = 3$ voters when $p < \bar{p}$, is $1 - F(\omega_v)(1 - p)/p$. Thus, for $p < \bar{p}$ committee size is irrelevant, and a small committee with the minimum number of just three voters generates the same probability of correct decision-making as a large electorate. Now let $p > \bar{p}$ and compare a committee with five or more voters to one with just three voters, for which the probability of correct decision-making is $1 - F(\omega_v)(1 - J_3(p))/(1 - p)$. Comparing the probabilities of correct decision-making across these two committee sizes, we find that the 'large' committee (with $n \geq 5$ voters) is superior to the three-member one iff:

$$1 - J_3(p) > \frac{(1-p)^2}{p} \Leftrightarrow \frac{(2p-1)(1+p)(1-p)^2}{p} > 0. \tag{4}$$

As we assume that $p \in (1/2, 1)$, the inequality on the right-hand side of (4) always holds. Thus, any 'large' committee with five or more voters maximizes the probability of correct decision-making when $p \in (\bar{p}, 1 - F(\omega_v))$. □

This result shows that in settings with a high chance of preference misalignment between expert and voters, the smallest committee size needed to maximize the probability of a correct decision increases with the precision p of voters' private signals.

5 Conclusion

In this paper we have studied the effect of the transmission of public information through a persuasion strategy chosen by a biased expert to partially informed voters. We show that public persuasion never limits information aggregation if the precision of voters' private signals is low. Otherwise, public persuasion will hurt information aggregation in large committees. This is because the information conveyed through the equilibrium persuasion strategy overpowers voters' private information of and invariably makes them vote for a particular alternative. In contrast, without persuasion voters will vote according to their private signals so that the probability of the correct decision increases with the size of the electorate. Thus, a lack of expert advice can actually improve information aggregation in large constituencies. A key insight of this paper is that a similar issue arises even in small constituencies, even though not for all constellations of the model parameters.

Possible extensions to our model include alternative voting rules (such as approval voting or cumulative voting), and three or more alternatives. It would also be quite quite natural to introduce multiple experts with either similar or conflicting biases, and examine the effect of their communication on the electorate's chances of making the right decision. We reserve these for future research.

6 Appendix

6.1 Proof of Proposition 1

In order to establish the result in Prop. 1, we ask which voting strategies $v : S \to A$ constitute a symmetric equilibrium. Due to the binary nature of the set of alternatives and the signal realizations, there are only four voting strategies to consider:

Voting strategy 1: always vote for X regardless of signal.

Suppose $v(X) = X$ and $v(Y) = X$. In this case, $n_X(\boldsymbol{v}(\boldsymbol{s}_{-i})) = n-1$ for all $\boldsymbol{s}_{-i} \in S^{n-1}$ and therefore $\Pi_i(v) = \varnothing$. This means that voter i is never pivotal. Whenever he is not pivotal, we assume that voter i votes on the basis of his private signal s_i as if his vote alone determined the outcome. Computing voter i's interim utility difference from voting for X rather than Y given his private signal s_i yields:

$$\int_{\Omega} \left(u(X,\omega) - u(Y,\omega)\right) f(\omega|s_i) d\omega = (\bar{u} - \underline{u}) \left(2F(\omega_v|s_i) - 1\right).$$

Therefore, voter i votes for X if $F(\omega_v|s_i) \geq 1/2$, and otherwise he votes for Y. In particular:

- if $s_i = X$, we have: $F(\omega_v|X) = \frac{pF(\omega_v)}{pF(\omega_v)+(1-p)(1-F(\omega_v))}$.

  Thus, voter i with private signal $s_i = X$ votes for X if $p \geq 1 - F(\omega_v)$, and otherwise he votes for Y.

- if $s_i = Y$, we have: $F(\omega_v|Y) = \frac{(1-p)F(\omega_v)}{(1-p)F(\omega_v)+p(1-F(\omega_v))}$.

  Thus, voter i with private signal $s_i = Y$ votes for X if $p \leq F(\omega_v)$, and otherwise he votes for Y.

Summary on voting strategy 1: For a symmetric voting equilibrium where every voter votes for X regardless of his private signal, we need parameter values p and $F(\omega_v)$ s.t. $p \in [1 - F(\omega_v), F(\omega_v)]$ which, in turn, requires a setting with a *high* likelihood of agreement: $F(\omega_v) > 1/2$.

Voting strategy 2: always vote for Y regardless of signal.

Suppose $v(X) = Y$ and $v(Y) = Y$. In this case, $n_X(\boldsymbol{v}(\boldsymbol{s}_{-i})) = 0$ for all $\boldsymbol{s}_{-i} \in S^{n-1}$ and therefore $\Pi_i(v) = \varnothing$. This means that voter i is never pivotal. The remainder of this case is analogous to that of voting strategy 1, and we refer the reader to the expressions for $F(\omega_v|X)$ and $F(\omega_v|Y)$ given there.

Summary on voting strategy 2: For a symmetric voting equilibrium where every voter votes for Y regardless of his private signal, we need parameter values p and $F(\omega_v)$ s.t. $p \in (F(\omega_v), 1 - F(\omega_v))$ which, in turn, requires a setting with a *low* likelihood of agreement: $F(\omega_v) < 1/2$.

Voting strategy 3: vote in line with signal.

Suppose $v(X) = X$ and $v(Y) = Y$. In this case, $\Pi(v)$ is nonempty as voter i is pivotal whenever $(n-1)/2$ other voters get an X-signal, while the rest get a Y-signal. Appealing to the observation made in Section 3.1 that $F(\omega_v|s_i, \boldsymbol{s}_{-i}) = F(\omega_v|s_i)$ for every $\boldsymbol{s}_{-i} \in \Pi_i(v)$, we can again refer the reader to the expressions for $F(\omega_v|X)$ and $F(\omega_v|Y)$ given above in the case of voting strategy 1.

Summary on voting strategy 3: For a symmetric voting equilibrium where every voter votes in line with his private signal, we need parameter values p and $F(\omega_v)$ s.t. either there is *high* likelihood of agreement (i.e. $F(\omega_v) > 1/2$) *and* $p > F(\omega_v)$, or there is *low* likelihood of agreement (i.e. $F(\omega_v) < 1/2$) *and* $p \geq 1 - F(\omega_v)$.

Voting strategy 4: vote contrary to signal.

Suppose $v(X) = Y$ and $v(Y) = X$. In this case, $\Pi(v)$ is nonempty as voter i is pivotal whenever $(n-1)/2$ other voters get an X-signal, while the rest get a Y-signal. As $F(\omega_v|s_i, \boldsymbol{s}_{-i}) = F(\omega_v|s_i)$ for every $\boldsymbol{s}_{-i} \in \Pi_i(v)$, we refer the reader to the expressions for $F(\omega_v|X)$ and $F(\omega_v|Y)$ given above in the case of voting strategy 1.

Summary on voting strategy 4: For a symmetric voting equilibrium where every voter votes contrary to his signal, we need parameter values p and $F(\omega_v)$ s.t. $p < 1 - F(\omega_v)$ and $p \leq F(\omega_v)$. However, as $p > 1/2$ by assumption, these two conditions cannot hold simultaneously: if the first strict inequality holds, then $F(\omega_v) < 1/2$. However, this contradicts the second weak inequality. Similarly, if the second weak inequality holds, then $F(\omega_v) > 1/2$, which contradicts the first strict inequality. We can therefore conclude that the voting strategy where every voter votes contrary to his private signal *cannot* be a symmetric equilibrium of the voting game. □

6.2 Proof of Proposition 2

Step 1: First consider the case $1/2 < p < F(\omega_v)$. From Prop. 1 it follows that when the expert's persuasion strategy is Ω^1 so that no information is provided, then $v(\Omega^1, s_i) = X$ for all $i \in I$ and all $s_i \in S$. As every voter votes for the expert's preferred alternative regardless of his private signal, the expert has no incentive to deviate any other persuasion strategy, making (Ω^1, v) an equilibrium of the game.

Step 2: Next consider the case $1/2 < F(\omega_v) < p$.

Step 2.1: Suppose the expert uses a binary persuasion strategy Ω^2 where $\Omega_1^2 = [0, \omega']$ and $\Omega_2^2 = (\omega', 1]$ with $\omega_v < \omega' \leq 1$. Furthermore, the threshold ω' that defines this binary strategy is chosen so as to induce the following voting behavior: $v(\Omega_1^2, s_i) = X$ and $v(\Omega_2^2, s_i) = Y$ for all $s_i \in S$.

To see how the equilibrium threshold ω' is determined, consider the voting behavior of a representative voter following the public message Ω_1^2 from the expert. Given that all voters other than i vote for X regardless of their private signals, voter i is *not* pivotal. In this case, we assume voter i votes on the basis of his private signal s_i and the expert's message Ω_1^2 as if his vote alone determined the outcome. Computing voter i's interim utility difference from voting for X rather than Y given his private signal s_i yields:

$$\begin{aligned}
& \int_{\Omega_1^2} (u(X,\omega) - u(Y,\omega)) f(\omega|s_i, \Omega_1^2) d\omega \\
= \quad & (\bar{u} - \underline{u}) \left(2F(\omega_v|s_i, \Omega_1^2) - 1 \right).
\end{aligned}$$

Therefore, voter i votes for X if $F(\omega_v|s_i, \Omega_1^2) \geq 1/2$, and otherwise he votes for Y. In particular, if $s_i = X$, we obtain by Bayes' Rule:

$$\begin{aligned}
F(\omega_v|X, \Omega_1^2) &= \frac{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v|\Omega_1^2)}{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v|\Omega_1^2) + \mathbb{P}[X|\omega > \omega_v](1 - F(\omega_v|\Omega_1^2))} \\
&= \frac{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v)/F(\omega')}{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v)/F(\omega') + \mathbb{P}[X|\omega > \omega_v](F(\omega') - F(\omega_v))/F(\omega')} \\
&= \frac{pF(\omega_v)}{pF(\omega_v) + (1-p)(F(\omega') - F(\omega_v))}.
\end{aligned}$$

Thus, voter i with private signal $s_i = X$ votes for X if $F(\omega') \leq F(\omega_v)/(1-p)$, and otherwise he votes for Y.

If, instead, voter i's private signal is $s_i = Y$, we obtain by Bayes' Rule:

$$\begin{aligned}
F(\omega_v|Y) &= \frac{\mathbb{P}[Y|\omega \leq \omega_v] F(\omega_v|\Omega_1^2)}{\mathbb{P}[Y|\omega \leq \omega_v] F(\omega_v|\Omega_1^2) + \mathbb{P}[Y|\omega > \omega_v](1 - F(\omega_v|\Omega_1^2))} \\
&= \frac{\mathbb{P}[Y|\omega \leq \omega_v] \frac{F(\omega_v)}{F(\omega')}}{\mathbb{P}[Y|\omega \leq \omega_v] \frac{F(\omega_v)}{F(\omega')} + \mathbb{P}[Y|\omega > \omega_v] \frac{F(\omega') - F(\omega_v)}{F(\omega')}} \\
&= \frac{(1-p)F(\omega_v)}{(1-p)F(\omega_v) + p(F(\omega') - F(\omega_v))}.
\end{aligned}$$

Thus, if $s_i = Y$ voter i votes for X if $F(\omega') \leq F(\omega_v)/p$, and otherwise he votes for Y.

Now consider voter i's voting behavior following the public message Ω_2^2 from the expert. Given that all voters other than i vote for Y regardless of their private signals, voter i is *not* pivotal. Casting his vote as if it alone determined the outcome, voter i will vote for Y as $F(\omega_v|s_i, \Omega_2^2) = 0$.

To summarize: As $p > 1/2$, it follows that given a message of $\Omega_1^2 = [0, \omega']$ by the expert, with $\omega' \in (\omega_v, 1]$ s.t. $F(\omega') \leq F(\omega_v)/p$, voter i votes for X regardless of his signal. Given a message of $\Omega_2^2 = (\omega', 1]$ with any $\omega' \in (\omega_v, 1]$, voter i votes for Y regardless of his signal.

We now look for the expert's optimal threshold value $\omega' \in (\omega_v, 1]$ s.t. $F(\omega') \leq F(\omega_v)/p$. Note that the expert's expected payoff from such a binary persuasion strategy is:

$$\begin{aligned}
&\int_\Omega \sum_{\boldsymbol{s}\in S^n} \mathbb{P}[\boldsymbol{s}|\omega] u_m(\delta(\boldsymbol{v}(\Omega^2,\boldsymbol{s})),\omega) f(\omega) d\omega \\
&= \int_0^{\omega'} \bar{u}_m f(\omega) d\omega + \int_{\omega'}^1 \underline{u}_m f(\omega) d\omega \\
&= \underline{u}_m + (\bar{u}_m - \underline{u}_m) F(\omega'). \qquad \text{(A.1)}
\end{aligned}$$

It is therefore optimal for the expert to choose the highest possible threshold value for ω', which is the one defined implicitly by the equation $F(\omega^*) = F(\omega_v)/p$.

Step 2.2: Suppose the expert uses a ternary persuasion strategy Ω^3 with $\Omega_1^3 = [0,\alpha)$, $\Omega_2^3 = [\alpha,\beta]$, and $\Omega_3^3 = (\beta,1]$, where $\alpha \in [0,\omega_v)$ and $\beta \in (\omega_v,1]$. Furthermore, the thresholds α and β that define this ternary strategy are chosen so as to induce the following voting behavior: $v(\Omega_1^3,s_i) = X$ for all $s_i \in S$, $v(\Omega_2^3,s_i) = s_i$, and $v(\Omega_3^2,s_i) = Y$ for all $s_i \in S$.

To see how the equilibrium thresholds α and β are determined, we start by considering the voting behavior of a representative voter. Recall that for any $t = 1,2,3$, voter i will vote for X if $F(\omega_v|s_i,\Omega_t^3) \geq 1/2$, and otherwise he votes for Y. It is straightforward to see that after a public message of Ω_1^3 from the expert, we have $F(\omega_v|s_i,\Omega_1^3) = 1$ regardless of the realization of the private signal. So while voter i is not pivotal as all other voters vote for X regardless of their private signals, it is optimal for voter i to follow the same strategy given our assumption that he will vote as if his ballot alone determines the outcome.

Similarly, following a public message of Ω_3^3 from the expert, we have $F(\omega_v|s_i,\Omega_3^3) = 0$ regardless of the realization of the private signal. So while voter i is not pivotal as all other voters vote for Y regardless of their private signals, it is optimal for voter i to follow the same strategy given our assumption that he will vote as if his ballot alone determines the outcome.

Finally, following the public message Ω_2^3 from the expert, all voters other than i vote according to their respective signals. Therefore, voter i is pivotal for any signal-profile $\boldsymbol{s}_{-i} \in \Pi_i(v)$. In particular, if $s_i = X$, we obtain by Bayes' Rule:

$$\begin{aligned}
F(\omega_v|X,\Omega_2^3) &= \frac{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v|\Omega_2^3)}{\mathbb{P}[X|\omega \leq \omega_v] F(\omega_v|\Omega_2^3) + \mathbb{P}[X|\omega > \omega_v](1 - F(\omega_v|\Omega_2^3))} \\
&= \frac{\mathbb{P}[X|\omega \leq \omega_v] \frac{F(\beta)-F(\omega_v)}{F(\beta)-F(\alpha)}}{\mathbb{P}[X|\omega \leq \omega_v] \frac{F(\beta)-F(\omega_v)}{F(\beta)-F(\alpha)} + \mathbb{P}[X|\omega > \omega_v] \frac{F(\omega_v)-F(\alpha)}{F(\beta)-F(\alpha)}} \\
&= \frac{p(F(\beta) - F(\omega_v))}{p(F(\beta) - F(\omega_v)) + (1-p)(F(\omega_v) - F(\alpha))}.
\end{aligned}$$

Thus, voter i with private signal $s_i = X$ votes for X if $F(\beta) \geq F(\omega_v) + \frac{1-p}{p}(F(\omega_v) - F(\alpha))$, and otherwise he votes for Y.

If, instead, voter i's private signal is $s_i = Y$, we obtain by Bayes' Rule:

$$\begin{aligned} F(\omega_v|Y) &= \frac{\mathbb{P}[Y|\omega \leq \omega_v] F(\omega_v|\Omega_2^3)}{\mathbb{P}[Y|\omega \leq \omega_v] F(\omega_v|\Omega_2^3) + \mathbb{P}[Y|\omega > \omega_v](1 - F(\omega_v|\Omega_2^3))} \\ &= \frac{\mathbb{P}[Y|\omega \leq \omega_v] \frac{F(\beta)-F(\omega_v)}{F(\beta)-F(\alpha)}}{\mathbb{P}[Y|\omega \leq \omega_v] \frac{F(\beta)-F(\omega_v)}{F(\beta)-F(\alpha)} + \mathbb{P}[Y|\omega > \omega_v] \frac{F(\omega_v)-F(\alpha)}{F(\beta)-F(\alpha)}} \\ &= \frac{(1-p)(F(\beta)-F(\omega_v))}{(1-p)(F(\beta)-F(\omega_v)) + p(F(\omega_v)-F(\alpha))}. \end{aligned}$$

Thus, if $s_i = Y$ voter i votes for Y if $F(\beta) < F(\omega_v) + \frac{p}{1-p}(F(\omega_v) - F(\alpha))$, and otherwise he votes for X.

To summarize: As $p > 1/2$ it is a best response for voter i to vote in line with his signal when message Ω_2^3 is conveyed by the expert and the other voters vote in line with their private signals.

We now look for the expert's optimal threshold values α and β. Note that the expert's expected payoff from the above ternary persuasion strategy is:

$$\begin{aligned} &\int_\Omega \sum_{\boldsymbol{s}\in S^n} \mathbb{P}[\boldsymbol{s}|\omega] u_m(\delta(\boldsymbol{v}(\Omega^3, \boldsymbol{s})), \omega) f(\omega) d\omega \\ &= \int_0^\alpha \bar{u}_m f(\omega) d\omega + \int_\alpha^{\omega_v} (J_n(p)(\bar{u}_m - \underline{u}_m) + \underline{u}_m) f(\omega) d\omega \\ &+ \int_{\omega_v}^\beta (\bar{u}_m - J_n(p)(\bar{u}_m - \underline{u}_m)) f(\omega) d\omega + \int_\beta^1 \underline{u}_m f(\omega) d\omega \\ &= \underline{u}_m + (\bar{u}_m - \underline{u}_m)\left[(F(\alpha) + F(\beta))(1 - J_n(p)) - F(\omega_v)(1 - 2J_n(p))\right], \end{aligned} \quad \text{(A.2)}$$

where $J_n(p) = \sum_{j=\frac{n+1}{2}}^n \binom{n}{j} p^j (1-p)^{n-j}$. As $\bar{u}_m > \underline{u}_m$ and $J_n(p) < 1$, the expert maximizes his expected payoff from the ternary persuasion strategy by choosing the thresholds α and β so as to maximize $F(\alpha) + F(\beta)$ subject to the constraints: (i) $F(\beta) \geq F(\omega_v) + \frac{1-p}{p}(F(\omega_v) - F(\alpha))$, and (ii) $F(\beta) < F(\omega_v) + \frac{p}{1-p}(F(\omega_v) - F(\alpha))$.

In order to obtain the solution to the expert's optimization problem, note that the threshold α determines the interval boundaries constraining the choice of threshold β. In particular, if α approaches its maximum admissible value ω_v, then $\beta = \omega_v$ and the ternary persuasion strategy collapses to a binary one with suboptimal threshold $\omega' = \omega_v$. Instead, we set the threshold β equal to its maximum admissible value: $\beta^* = 1$. With this value, constraint (ii) yields the following upper bound on α: $F(\alpha) < (p - (1 - F(\omega_v)))/p$. As $p > 1/2 > 1 - F(\omega_v)$, this upper bound is positive and the optimal threshold α^* is determined implicitly by the following equation: $F(\alpha^*) = 1 - ((1 - F(\omega_v))/p)$.

Step 2.3:Having characterized the optimal thresholds α^* and β^*, we can now compute the difference in the expert's expected payoffs from the optimal binary and ternary persuasion strategies:

$$\begin{aligned} &\int_\Omega \sum_{\boldsymbol{s}\in S^n} \mathbb{P}[\boldsymbol{s}|\omega] \left(u_m(\delta(\boldsymbol{v}(\Omega^3, \boldsymbol{s})), \omega) - u_m(\delta(\boldsymbol{v}(\Omega^2, \boldsymbol{s})), \omega)\right) f(\omega) d\omega \\ &= \frac{(\bar{u}_m - \underline{u}_m)(2p-1)(1-F(\omega_v))}{p} \left(1 - J_n(p) - \frac{F(\omega_v)/(1-F(\omega_v))}{(p/(1-p)) - 1}\right). \end{aligned} \quad \text{(A.3)}$$

If the payoff difference in (A.3) is negative, then the binary persuasion strategy is optimal for the expert, and if it is positive, then the ternary persuasion strategy is optimal. The sign of the payoff difference in (A.3) is determined by the sign of the expression in the large brackets. Note that this expression is decreasing in the value of $J_n(p)$. Here we avail ourselves of the following result that is derived in the proof of Lemma 2 in Karotkin and Paroush (2003):

Lemma (?). $J_n(p)$ is increasing in n. In particular: $J_{n+2}(p) - J_n(p) = p(2p-1)\binom{n}{\frac{n-1}{2}}p^{\frac{n-1}{2}}(1-p)^{\frac{n+1}{2}}$.

By Karotkin and Paroush's lemma it follows that if the expression in large brackets in (A.3) is negative for $n = 3$, then it will also be negative for all $n > 3$. We now show by contradiction that this expression is negative for $n = 3$. Note that $J_3(p) = 3p^2(1-p) + p^3$. Suppose now that the expression in large brackets in (A.3) is non-negative for $n = 3$:

$$1 - J_3(p) \geq \frac{F(\omega_v)/(1-F(\omega_v))}{(p/(1-p)) - 1} \Leftrightarrow p\,(4p(1-p)+1) \geq 1 + \frac{F(\omega_v)}{1-F(\omega_v)}$$

It is easy to verify that this latter inequality cannot hold: as $F(\omega_v) > 1/2$, the expression on the right-hand side of the inequality is at least 2, while the polynomial on the left-hand side is at most $(1/27)(17 + 7^{3/2}) \approx 1.3156$. We can therefore conclude that the difference in the expert's expected payoffs from the optimal ternary and binary persuasion strategies is negative for all $n \geq 3$.

Finally, note that in an equilibrium with a binary persuasion strategy, the expert's expected payoff is increasing in ω', and the maximum value of ω' under this type of persuasion strategy is attained when the condition $F(\omega') = F(\omega_v)/p$ holds. When $F(\omega') > F(\omega_v)/p$ then $F(\omega_v|Y) < 1/2$ and the equilibrium involves a ternary persuasion strategy with $\alpha = 0$ and $\beta = \omega'$. From Step 2.2 above it is easy to see that such a persuasion strategy involves a lower expected payoff for the expert than the equilibrium under optimal binary persuasion.[6] This proves part (b) of the proposition and concludes the proof. □

6.3 Proof of Proposition 3

First consider the performance of the binary persuasion strategies defined above in the proof of Prop. 1. Just as in the case of a high likelihood of agreement, the optimal binary persuasion strategy in the case of a high likelihood of ex post conflict features a threshold ω^* defined implicitly by $F(\omega^*) = F(\omega_v)/p$, which yields the expert an expected payoff of $\underline{u}_m + (\bar{u}_m - \underline{u}_m)F(\omega_v)/p$.

Next, consider the performance of the ternary persuasion strategies defined in the proof of Prop. 1. In the present case of a high likelihood of ex post conflict, we have to distinguish the following scenarios:

Suppose that $p > 1 - F(\omega_v)$. In this case, the expert's problem of maximizing his expected payoff from the ternary persuasion strategy is identical to the one in the proof of Prop. 1 (see Step 2.2), so that items 1.(a) and 1.(b) of Prop. 2 are obtained immediately by verifying that the expert's difference in expected payoff from the optimal ternary and binary persuasion strategies in equation (A.3) is negative.

[6] There is only one other possible class of binary persuasion strategies Ω^2, namely the one with $\Omega_1^2 = [0, \omega')$ and $\Omega_2^2 = [\omega', 1]$, where $\omega' < \omega_v$ s.t. $v_i(\Omega_1^2, s_i) = X$ and $v_i(\Omega_2^2, s_i) = Y$ for every $s_i \in S$. This class clearly yields a lower expected payoff for the expert than the optimal binary persuasion strategy in Step 2.1, and therefore can never be optimal for the expert.

Now suppose that $p < 1 - F(\omega_v)$. In this case, the expert's problem of maximizing his expected payoff from the ternary persuasion strategy subject to the constraints $F(\beta) \geq F(\omega_v) + \frac{1-p}{p}(F(\omega_v) - F(\alpha))$ and $F(\beta) < F(\omega_v) + \frac{p}{1-p}(F(\omega_v) - F(\alpha))$ yields optimal threshold values $\alpha^* = 0$ and β^* defined implicitly by $F(\beta^*) = F(\omega_v)/(1-p)$. By substituting these optimal threshold values into the equation for the expert's expected payoff from the ternary persuasion in (A.2), we can obtain the following expected payoff difference:

$$\int_\Omega \sum_{\boldsymbol{s}\in S^n} \mathbb{P}[\boldsymbol{s}|\omega] \left(u_m(\delta(\boldsymbol{v}\left(\Omega^3, \boldsymbol{s}\right)), \omega) - u_m(\delta(\boldsymbol{v}\left(\Omega^2, \boldsymbol{s}\right)), \omega)\right) f(\omega) d\omega$$
$$= F(\omega_v)(\bar{u}_m - \underline{u}_m) \left[J_n(p)p(2p-1) - p^2 - p + 1\right] / p(1-p).$$

Label as $\eta(n,p)$ the term in square brackets: $\eta(n,p) \equiv J_n(p)p(2p-1) - p^2 - p + 1$. Note that $\eta(5,1/2) = 1/4$, $\eta(5,1) = 0$, and that $\eta(5,p) = 0$ has no solution in $p \in (1/2,1)$. Thus, $\eta(5,p) > 0$ for all $p \in (1/2,1)$. As $p > 1/2$ it follows that $\eta(n,p)$ is increasing in $J_n(p)$ which, by Claim 1, is increasing in n. This implies that $\eta(n,p) > 0$ for all $n \geq 5$, which establishes the result in part 2.(a) of Prop. 2.

To prove part 2.(b), set $n = 3$. Note that $\eta(3,1/2) = 1/4$, $\eta(3,1) = 0$, and that $\eta(3,p) = 0$ has a unique solution in $p \in (1/2,1)$ given by $\bar{p} = ((27 - 3\sqrt{78})^{1/3} + (3\sqrt{78} + 27)^{1/3})/6 \approx 0.76$. This shows that for all $p \in (1/2, \bar{p})$, we have $\eta(3,p) > 0$, while for all $p \in (\bar{p}, 1)$, we have $\eta(3,p) < 0$. This proves part 2.(b) and completes the proof. □

6.4 Proof of Lemma 1

Recall the function G_n which was introduced in (2) in the main text. Defined formally:

$$G_n : [0,1] \to \mathbb{R}_+, p \mapsto G_n(p) = \frac{p}{1-p} \left(1 - \sum_{j=(n+1)/2}^{n} \binom{n}{j} p^j (1-p)^{n-j}\right).$$

To establish Lemma 1, we appeal to Weierstrass' extreme value theorem, according to which the function G_n must attain at least one maximum and at least one minimum in $[0,1]$. Obviously, the points $p_1^* = 0$ and $p_2^* = 1$ constitute global minima as G_n can take only non-negative values. We now argue that p_1^* and p_2^* are the only (local and global) minimum points of G_n, and that there is a unique interior global maximum at some point $p_3^* \in (0,1/2)$. These results will be established by means of two claims, the proofs of which are given here in the Appendix after the proof of Lemma 1.

Claim 1. *For all $n \geq 7$, the function $G_n(p)$ is strictly decreasing at $p = 1/2$: $G_n'(1/2) < 0$.*

Claim 2. *For $p < \frac{1}{2} + \frac{1}{n+1}$, the function $G_n(p)$ is strictly* concave *in the neighborhood of any critical point p^*: $G_n''(p^*) < 0$; for all $p > \frac{1}{2} + \frac{1}{n+1}$, the function $G_n(p)$ is strictly* convex *in the neighborhood of any critical point p^*: $G_n''(p^*) > 0$.*

Now let $n \geq 7$. By the extreme value theorem, we know that there is at least one global maximum point of G_n. We establish indirectly that this global maximum occurs at some value strictly below 1/2. Assume first that there is an interior local maximum $\tilde{p} \in (\frac{1}{2}, \frac{1}{2} + \frac{1}{n+1})$. Given that $G_n'(1/2) < 0$ (by Claim 1), a necessary condition for a local interior maximum within this subinterval is the existence of a turning-point at a local interior minimum $\hat{p} \in (\frac{1}{2}, \tilde{p})$. At such a point, it holds that $G_n''(\hat{p}) > 0$, which constitutes a contradiction to Claim 2. Thus, there is no subinterval $[\hat{p}, \tilde{p}] \subset (\frac{1}{2}, \frac{1}{2} + \frac{1}{n+1})$ on which the function G_n is increasing, which allows us to conclude that G_n is strictly decreasing on $[1/2, \frac{1}{2} + \frac{1}{n+1})$.

Next, by Claim 2 it follows immediately that there is also no interior local maximum $\tilde{p} \in (\frac{1}{2} + \frac{1}{n+1}, 1)$. This implies furthermore that there cannot be an interior local minimum of G_n in this range. To see this, suppose there is such a minimum at some point $\hat{p} \in (\frac{1}{2} + \frac{1}{n+1}, 1)$. As $G_n(1) = 0$, the function G_n must reach a turning-point at a local interior maximum $\tilde{p} \in (\hat{p}, 1)$ before monotonically decreasing until it reaches the value 0 at $p = 1$. But by Claim 2 we know that no such maximum $\tilde{p}$ exists, and consequently there is no subinterval $[\hat{p}, \tilde{p}] \subset (\frac{1}{2} + \frac{1}{n+1}, 1)$ on which the function G_n is increasing. We conclude that G_n is strictly decreasing on $(\frac{1}{2} + \frac{1}{n+1}, 1)$. Finally, $\lim_{p \to (\frac{1}{2} + \frac{1}{n+1})^-} G_n(p) = G_n(\frac{1}{2} + \frac{1}{n+1}) = \lim_{p \to (\frac{1}{2} + \frac{1}{n+1})^+} G_n(p)$ by continuity of G_n. This implies that G_n is strictly decreasing on $[1/2, 1)$ for all $n \geq 7$. $\square$

6.5 Proof of Claim 1

In order to show that the function $G_n(p) = \frac{p}{1-p} - \frac{p}{1-p} J_n(p)$ is strictly decreasing at $p = 1/2$, we begin by computing the derivative $G_n'(p)$:

$$G_n'(p) = \frac{1}{(1-p)^2} - \frac{J_n(p)}{(1-p)^2} - \frac{pJ_n'(p)}{(1-p)}.$$

As $J_n(p) = \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} p^j (1-p)^{n-j}$ is an additive function, it is easy to verify that:

$$J_n'(p) = \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} p^j (1-p)^{n-j} \left(\frac{j - np}{p(1-p)} \right).$$

Thus, we can write:

$$G_n'(p) = \frac{1}{(1-p)^2} - \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} p^j (1-p)^{n-j} \left(\frac{1 + j - np}{(1-p)^2} \right).$$

Evaluating this derivative at $p = 1/2$ yields:

$$G_n'(1/2) = 4 \left(1 - \left(\frac{1}{2} \right)^{n+1} \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2 + 2j - n) \right).$$

We want to establish that $G_n'(1/2) < 0$ for all $n \geq 7$, which is the case iff:

$$2^{n+1} < \sum_{j=\frac{n+1}{2}}^{n} \binom{n}{j} (2 + 2j - n). \tag{A.4}$$

Now note that $\sum_{j=0}^{n}\binom{n}{j}=2^n$, and since n is odd, we also have $\sum_{j=\frac{n+1}{2}}^{n}\binom{n}{j}=(1/2)\sum_{j=0}^{n}\binom{n}{j}$. Using these identities, we may express (A.4) as:

$$2^n<\sum_{j=\frac{n+1}{2}}^{n}\binom{n}{j}(2j-n)\Leftrightarrow 2\sum_{j=\frac{n+1}{2}}^{n}\binom{n}{j}<\sum_{j=\frac{n+1}{2}}^{n}\binom{n}{j}(2j-n)$$

$$\Leftrightarrow 0<\sum_{j=\frac{n+1}{2}}^{n}\binom{n}{j}(2(j-1)-n). \tag{A.5}$$

A sufficient condition for the inequality in (A.5) to hold is that the first three terms of the summation on the right-hand side add up to a strictly more than zero. All remaining terms in the summation (i.e. those where $j\geq 3+(n+1)/2$) are strictly positive and thereby only increase the value obtained by adding up the first three terms):

$$\binom{n}{\frac{n+3}{2}}+3\binom{n}{\frac{n+5}{2}}-\binom{n}{\frac{n+1}{2}}>0. \tag{A.6}$$

Now recall the following identity: $\binom{n}{j+1}=\binom{n}{j}\frac{n-j}{j+1}$ for all $j=0,\ldots,n-1$. We can use this to express equivalently the sufficient condition in (A.6):

$$\binom{n}{\frac{n+1}{2}}\left[\frac{n-1}{n+3}\left(1+\frac{3(n-3)}{n+5}\right)-1\right]>0,$$

which is satisfied iff $3n^2-16n-11>0$. Ignoring for a moment that the function on the left-hand side of this inequality maps odd integers $n\geq 3$ into the reals, the reader can easily verify that it has an interior local minimum at $n^*\approx 2.6667$, and that it is strictly increasing for all $n>n^*$. Furthermore, evaluating this function at $n=5$ and $n=7$ (which yields values of -16 and 24, resp.) we can see immediately that the sufficient condition in (A.6) is satisfied for all odd integers $n\geq 7$, which proves that $G_n'(1/2)<0$ for all these n. □

6.6 Proof of Claim 2

For ease of notation, define $L_{n,j}(p)\equiv\binom{n}{j}p^j(1-p)^{n-j}$. Also let:

$$F_{n,r}(p)\equiv\sum_{j=0}^{r}\binom{n}{j}p^j(1-p)^{n-j}=\sum_{j=0}^{r}L_{n,j}(p).$$

As n is an odd integer, the integer $m=\frac{n+1}{2}$ is even. We can therefore express equivalently the function G_n (as defined in the text and at the start of the proof of Claim 1) as follows: $G_n(p)=(p/(1-p))F_{n,m-1}(p)$. In order to obtain the derivatives of this function $G_n(p)$, it is useful to note that:

$$L_{n,j}'(p)=n\left(L_{n-1,j-1}(p)-L_{n-1,j}(p)\right). \tag{A.7}$$

Thus, $F_{n,m-1}'(p)=\sum_{j=0}^{m-1}n\left(L_{n-1,j-1}(p)-L_{n-1,j}(p)\right)=-nL_{n-1,m-1}(p)$. We therefore obtain our desired derivative:

$$G_n'(p)=\frac{1}{(1-p)^2}F_{n,m-1}(p)-\frac{np}{1-p}L_{n-1,m-1}(p). \tag{A.8}$$

Differentiating (A.8) yields:

$$G_n''(p) = \frac{2}{(1-p)^3} F_{n,m-1}(p) - \frac{2n}{(1-p)^2} L_{n-1,m-1}(p) - \frac{np}{1-p} L'_{n-1,m-1}(p).$$

Using (A.7) we obtain: $L'_{n-1,m-1}(p) = (n-1)\left(L_{n-2,m-2}(p) - L_{n-2,m-1}(p)\right)$, and thus:

$$\begin{aligned} G_n''(p) = \; & \frac{2}{(1-p)^3} F_{n,m-1}(p) - \frac{2n}{(1-p)^2} L_{n-1,m-1}(p) \\ & - \frac{np(n-1)}{1-p}\left(L_{n-2,m-2}(p) - L_{n-2,m-1}(p)\right). \end{aligned} \tag{A.9}$$

Now recall that by Weierstrass' extreme value theorem, the function G_n has at least one global maximum in $[0,1]$. As the boundary points $p = 0$ and $p = 1$ are both global minima of G_n, any global maximum p^* must occur in the interior of $[0,1]$. Thus, any such p^* is a critical point of G_n that satisfies the first-order condition $G_n'(p^*) = 0$. Using (A.8) we can express the first-order condition as follows:

$$\frac{1}{(1-p^*)^2} F_{n,m-1}(p^*) - \frac{np^*}{1-p^*} L_{n-1,m-1}(p^*) = 0.$$

The second-order condition for a local maximum requires $G_n''(p^*) \leq 0$, so substituting the above first-order condition into (A.9) we can rearrange terms to express the second-order condition for an interior maximum at p^* as follows:

$$n!\,p^{m-1}(1-p)^{n-m-2}\left(p^*(1+n) - (1+m)\right) \leq 0 \Leftrightarrow p^* \leq (1+m)/(1+n).$$

Re-substituting $\frac{n+1}{2}$ for m finally yields the threshold $\frac{1}{2} + \frac{1}{n+1}$ in the statement of Claim 2. □

6.7 Proof of item 2.(b) of Proposition 4

To prove item 2.(b) of Prop. 4, we start by considering the case of $n = 3$ voters. The left-hand panel of Fig. 2 shows the graph of the function $G_3(p) = p + p^2 - 2p^3$. Note that $G_3'(1/2) = 1/2$, and that the first-order condition $G_3'(p) = -6p^2 + 2p + 1 = 0$ yields a unique critical point $p_3 = (1+\sqrt{7})/6 \approx 0.60763$. As $G_3''(p) < 0$ for all $p \in [1/2, 1]$, the function G_3 is strictly concave everywhere in this range, and so the point p_3 is the unique global maximum of G_3. The corresponding functional value is $G_3(p_3) = (7\sqrt{7} + 10)/54$, which we label as q_3. Note that since $q_3 \in (1/2, 1)$, it follows immediately that in settings where $F(\omega_v) > q_3$, expert persuasion is detrimental to information aggregation. Now suppose instead that $1/2 < F(\omega_v) < q_3$. As G_3 is strictly concave and $G_3'(p) < 1$ for all $p \in [1/2, 1]$, the equation $p + p^2 - 2p^3 = F(\omega_v)$ has two real-valued solutions $\hat{p}_3^F$ and $\tilde{p}_3^F$ with $F(\omega_v) < \hat{p}_3^F < \tilde{p}_3^F < 1$. For all $p \in (\hat{p}_3^F, \tilde{p}_3^F)$ we have $G_3(p) > F(\omega_v)$, and for all $p \notin [\hat{p}_3^F, \tilde{p}_3^F]$ we have $G_3(p) < F(\omega_v)$.

Next, consider the case of $n = 5$ voters. The right-hand panel of Fig. 2 shows the graph of the function $G_5(p) = p(1-p)^2(6p^2 + 3p + 1)$. Note that $G_5'(1/2) = 1/8$, and that the first-order condition $G_5'(p) = (1-p)(1 + 3p + 6p^2 - 30p^3) = 0$ yields a unique critical point $p_5 = (2 + (548 - 30\sqrt{290})^{1/3} + (548 + 30\sqrt{290})^{1/3})/30 \approx 0.51761$. Note that whilst it is tedious to compute the roots of the cubic function $G_5''(p) = 2((1+3p) - (2p^2(27 - 30p)))$ in order to ascertain the curvature of G_5, it is easy to verify that $G_5''(p) < 0$ for all $p \in [1/2, 3/5]$. To see this, note that both the functions $1 + 3p$ and $2p^2(27 - 30p)$ are strictly increasing for

$p \in [1/2, 3/5)$, and that the maximum value of the former function (i.e. 2.8 when $p = 3/5$) is strictly lower than the minimum value of the latter function (i.e. 6 when $p = 1/2$). Therefore, G_5 is strictly concave for all $p \in [1/2, 3/5]$. From this, we can conclude that the point p_5 is the unique global maximum of G_5.

Rather than incur the tedium of verifying directly that the corresponding functional value $q_5 \equiv G_5(p_5)$ lies strictly in the range $(1/2, 1)$, we simply argue that for *any n* it holds that $G_n(p) < 1$ for all $p \in [1/2, 2/3)$. The condition $G_n(p) < 1$ is equivalent to $(1 - J_n(p)) < (1 - p)/p$. Recall that $J_n(1/2) = 1/2$ and $J_n(1) = 1$. Thus, $(1 - J_n(p)) \in [0, 1/2]$ for all $p \in [1/2, 1]$. Now note that $(1 - p)/p$ strictly exceeds $1/2$ for all $p \in [1/2, 2/3)$, which immediately implies that $G_5(p_5) < 1$. We can therefore state that in settings where $F(\omega_v) > q_5$, expert persuasion is detrimental to information aggregation.

Finally, the fact that G_5 is strictly concave for all $p \in [1/2, 3/5]$, that $G_5'(p) < 1$ for all $p \in [1/2, 1]$, and that $G_5(3/5) = 1488/3125 \approx 0.47616 < 1/2$ implies immediately that in settings where $1/2 < F(\omega_v) < q_5$, the equation $G_5(p) = F(\omega_v)$ has two real-valued solutions $\hat{p}_5^F$ and $\tilde{p}_5^F$ with $F(\omega_v) < \hat{p}_5^F < \tilde{p}_5^F < 1$. For all $p \in (\hat{p}_5^F, \tilde{p}_5^F)$ we have $G_5(p) > F(\omega_v)$, and for all $p \notin [\hat{p}_5^F, \tilde{p}_5^F]$ we have $G_5(p) < F(\omega_v)$. □